

# Spectator Effects in the Decay $B \rightarrow K\gamma\gamma$

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We report the results of the first computation related to the study of the spectator effects in the rare decay mode  $B \rightarrow K\gamma\gamma$  within the framework of Standard Model. It is found that the account of these effects results in the enhancement factor for the short-distance reducible contribution to the branching ratio.

**Keywords:** Rare B decay

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## I. INTRODUCTION

The decays of B mesons provide us with a valuable tool for studying CP violation, testing the Standard Model and looking for new physics. In particular, it has become possible to study low-probability processes such as various rare decay modes of B mesons. Some of these decay rates have already been measured while other rare decays are expected to be observed in the future.

One process of the latter category is the decay  $B \rightarrow K\gamma\gamma$ . Previously, this decay mode and/or the related quark process  $b \rightarrow s\gamma\gamma$  have been studied in the papers [1, 2, 3, 4, 5, 6, 7] where both short-distance and long-distance contributions to the decay rate have been considered. In Ref. [3] it was pointed out that one of the difficulties of the theoretical analysis is the account of spectator effects in such decays. Spectator effects are those processes when one of the photons is emitted by the u or d quark which are part of the decaying B meson.

To the best of our knowledge, the contributions due to these effects have not been estimated before and such an estimate is the purpose of the present paper.

## II. SPECTATOR EFFECTS

The effective Hamiltonian is [2]

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu) O_i(\mu), \quad (2.1)$$

with

$$\begin{aligned} O_1 &= (\bar{s}_i c_j)_{V-A} (\bar{c}_j b_i)_{V-A}, \\ O_2 &= (\bar{s}_i c_i)_{V-A} (\bar{c}_j b_j)_{V-A}, \\ O_3 &= (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A} \\ O_4 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \end{aligned}$$

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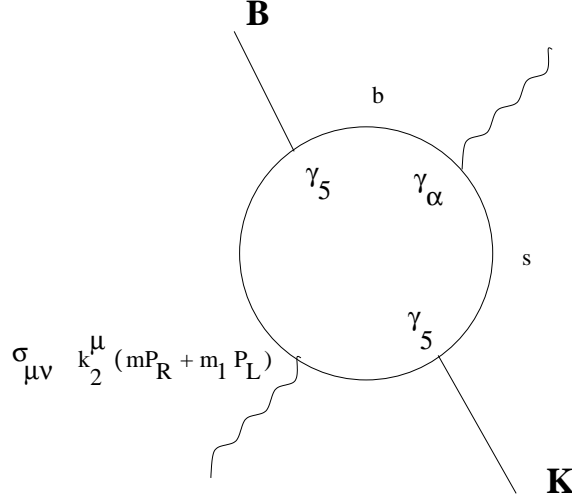


FIG. 1: Feynman diagram modelling the spectator effect in the decay  $B \rightarrow K\gamma\gamma$

$$\begin{aligned}
O_5 &= (\bar{s}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, \\
O_6 &= (\bar{s}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\
O_7 &= \frac{e}{16\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s P_L + m_b P_R) b_i F_{\mu\nu}, \quad \text{and} \\
O_8 &= \frac{g}{16\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s P_L + m_b P_R) T_{ij}^a b_j G_{\mu\nu}^a.
\end{aligned} \tag{2.2}$$

We model the spectator effects by the loop diagram in Fig. 1. The corresponding amplitude is

$$M_{AD} = C \times \int d^4 l \frac{\text{Tr}(\gamma_5 (\not{l} - \not{k}_1 - \not{k}_2 - \not{q} + m_2) \not{\epsilon}_1 (\not{l} - \not{k}_2 - \not{q}) + m_2) \gamma_5 ((\not{l} - \not{k}_2) + m_1) [-\not{k}_2, \not{\epsilon}_2] (i/2) (m P_R + m_1 P_L) (\not{l} + m))}{(l^2 - m^2)((l - k_1 - k_2 - q)^2 - m_2^2)((l - k_2 - q)^2 - m_2^2)((l - k_2)^2 - m_1^2)} \tag{2.3}$$

plus Bose-exchanged part, where

$$m \equiv m_b = 4.8 \text{ GeV}, \quad m_1 \equiv m_s = 0.15 \text{ GeV}, \quad m_2 \equiv m_d = 0.006 \text{ GeV} \tag{2.4}$$

are the quark masses;  $k_1, k_2$  and  $q$  are the 4-momenta of the two photons and the kaon, respectively. Neglecting the spectator effects the decay  $B \rightarrow K\gamma\gamma$  is similarly modelled by the diagrams in Fig. 2, 3. The amplitude is given by

$$M_{SD} = M_{2a} + M_{2b} \tag{2.5}$$

$$M_{2a} = C \int d^4 l \frac{\text{Tr}(\not{\epsilon}_1 (\not{l} + \not{k}_1 + m) \gamma_5 (\not{l} - \not{k}_2 - \not{q} + m_2) \gamma_5 (\not{l} - \not{k}_2 + m_1) [-\not{k}_2, \not{\epsilon}_2] (i/2) (m P_R + m_1 P_L) (\not{l} + m))}{(l^2 - m^2)((l - k_1 - k_2 - q)^2 - m_2^2)((l + k_1)^2 - m^2)((l - k_2)^2 - m_1^2)} \tag{2.6}$$

$$M_{2b} = C \int d^4 l \frac{\text{Tr}(\not{\epsilon}_1 (\not{l} + \not{k}_1 + m_1) [-\not{k}_2, \not{\epsilon}_2] (i/2) (m P_R + m_1 P_L) (\not{l} + \not{k}_1 + \not{k}_2 + m) \gamma_5 (\not{l} - \not{q} + m_2) \gamma_5 (\not{l} + m_1))}{(l^2 - m_1^2)((l + k_1 + k_2)^2 - m^2)((l + q)^2 - m_2^2)((l + k_2)^2 - m_1^2)} \tag{2.7}$$

plus Bose-exchanged parts.

All diagrams in Fig. 1, 2 and 3 are convergent. To calculate the amplitudes we have used two software packages: FeynCalc [8] and LoopTools [9].

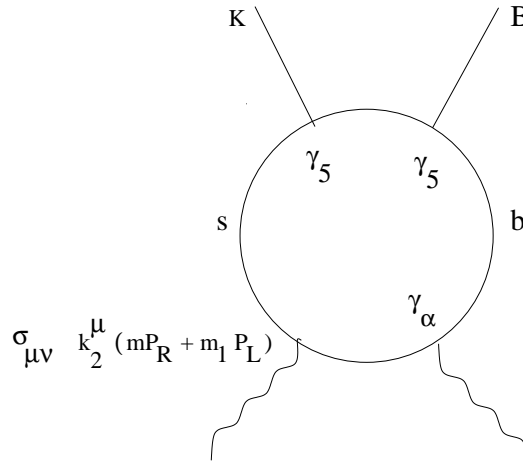


FIG. 2: Feynman diagram modelling the decay  $B \rightarrow K\gamma\gamma$  neglecting the spectator effects

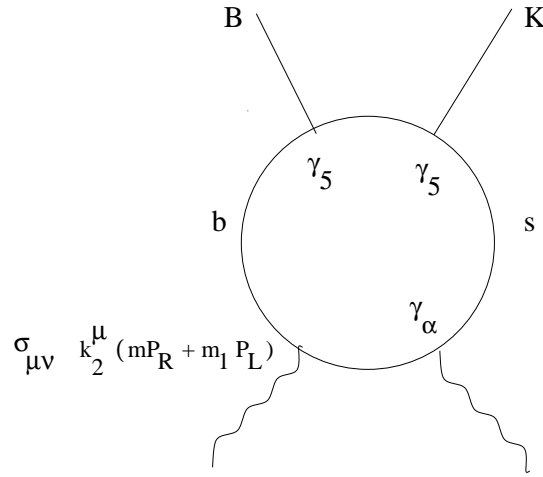


FIG. 3: Feynman diagram modelling the decay  $B \rightarrow K\gamma\gamma$  neglecting the spectator effects

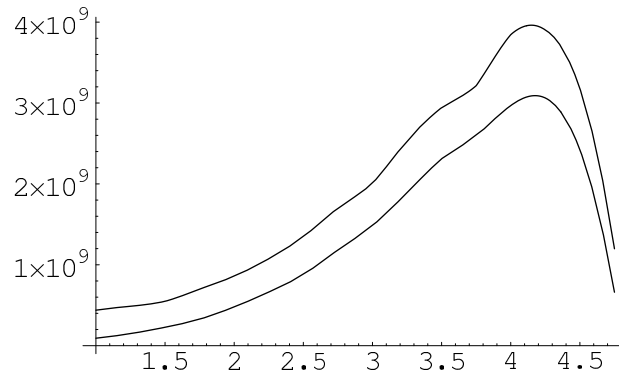


FIG. 4: The differential decay rates for the decay  $B^0 \rightarrow K^0\gamma\gamma$  proceeding through the amplitude  $M_{SD}$  (bottom curve) and through the amplitude  $M_{SD} + M_{AD}$  (top curve).

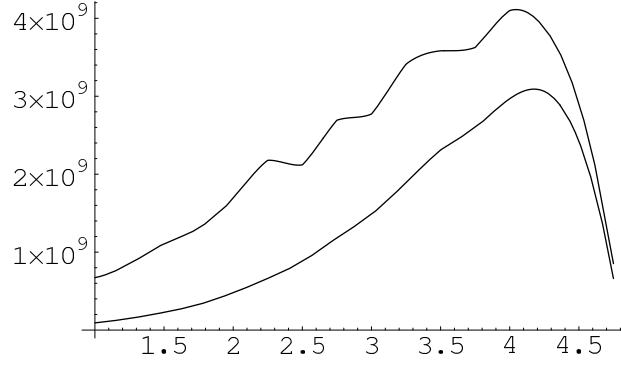


FIG. 5: The differential decay rates for the decay  $B^+ \rightarrow K^+ \gamma \gamma$  proceeding through the amplitude  $M_{SD}$  (bottom curve) and through the amplitude  $M_{SD} - 2M_{AD}$  (top curve). The factor (-2) comes from the ratio of the electric charges of the spectator quark and non-spectator quark.

TABLE I: Relative magnitudes of the spectator effects

	$B^0 \rightarrow K \gamma \gamma$	$B^+ \rightarrow K^+ \gamma \gamma$
r	1.38	1.65
$r_1$	1.32	1.38
$r_2$	1.32	1.36

To estimate the significance of the spectator effects we introduce the dimensionless ratio

$$r = \frac{\Gamma_{SD+AD}}{\Gamma_{SD}}. \quad (2.8)$$

Here,  $\Gamma_{SD+AD}$  is the contribution to the the  $B \rightarrow K \gamma \gamma$  decay rate including the spectator effects while  $\Gamma_{SD}$  is the the contribution neglecting the spectator effects:

$$\Gamma_i = \int \frac{1}{4m_B} |M_i|^2 \frac{d^3 k_1}{2\omega_1 (2\pi)^3} \frac{d^3 k_2}{2\omega_2 (2\pi)^3} \frac{d^3 q}{2\omega_K (2\pi)^3} (2\pi)^4 \delta(p_B - k_1 - k_2 - q). \quad (2.9)$$

It was found in Ref. [1] that contributions due to  $\eta'$ ,  $\eta_c$  and  $K^*$  were important. These contributions are not included here. Therefore, it is more informative to consider the following ratios with cuts:

$$r_1 = \frac{\Gamma_{SD+AD}(\sqrt{s_{\gamma\gamma}} > m_{\eta_c} + 2\Gamma_{\eta_c})}{\Gamma_{SD}(\sqrt{s_{\gamma\gamma}} > m_{\eta_c} + 2\Gamma_{\eta_c})} \quad (2.10)$$

and

$$r_2 = \frac{\Gamma_{SD+AD}(\sqrt{s_{\gamma\gamma}} > m_{\eta_c} + 2\Gamma_{\eta_c}, \sqrt{s_{K\gamma}} > m_{K^*} + 2\Gamma_{K^*})}{\Gamma_{SD}(\sqrt{s_{\gamma\gamma}} > m_{\eta_c} + 2\Gamma_{\eta_c}, \sqrt{s_{K\gamma}} > m_{K^*} + 2\Gamma_{K^*})}. \quad (2.11)$$

The calculation gives the results collected in Table 1 showing that the spectator effects contributions range from about 40 % in the case of neutral B mesons to about 60 % in the case of charged B mesons. Another way to represent the magnitude of the spectator effects is to plot the differential decay rates  $d\Gamma/d(\sqrt{s_{\gamma\gamma}})$  as functions of  $\sqrt{s_{\gamma\gamma}}$  (in GeV) through the amplitude  $M_{SD}$  and through the combination of the amplitudes  $M_{SD}$  and  $M_{AD}$ , see Fig. 4 and 5 (because we are only interested in the ratios of the decay rates, the units on the  $y$ -axis are arbitrary). From Table 1 we see that that the cuts induce a larger shift in  $r$  in the case of the decay  $B^+ \rightarrow K^+ \gamma \gamma$  as compared to the case of  $B^0 \rightarrow K \gamma \gamma$ . This is consistent with the fact that the gap between the two curves in Fig. 5 is larger than the corresponding gap in Fig. 4.

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